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SERIES AND BINOMIAL EXPANSIONS

SEQUENCE

Introduction!

→ A Sequence is a set of numbers stated in a definite order such that each term is obtained from the previous term according to some rule.
→ Each number in the sequence is called a term

Sequences: 1, 3, 5, 7, ---
4, 9, 16, 25, ---
2, 4, 6, 8, 10, ---

A Sequence can be ~~ed~~ endless continous (infinite) or can end at a certain term (definite)
E.g 1, 3, 5, 7, --- is infinite (does not end)
1, 2, 3, 4, ---, 50. is definite (ends at 50)

⇒ Any specific term in a sequence is called the nth term thus the ~~fir~~ 1st, 2nd, 3rd, --- last term can be written as

$$u_1, u_2, u_3, ---, u_n$$

∴ Hence the general unknown term is u_n.

⇒ We can find any term in the serie eg the 100th term without listing all the terms but using general formula.

⇒ We can also find the sum of any number of terms eg sum of the first 100 terms using formulae.

Example: Given the serie below

1, 3, 5, 7, 9, ---

⇒ We can find 100th term of the above serie
⇒ The sum of the first 4 terms is 1+3+5+7 = 16
⇒ We can also add the first 100 terms without listing all the terms.

ARITHMETIC PROGRESSION (A.P)

Consider the Serie 2, 5, 8, 11, 14, - - -

Note The first term is 2

The next term is obtained by adding 3 to the previous term hence there is a common difference of 3 accross the sequence.

The n^{th} term of A.P

Suppose the first term is denoted by a and the common difference is denoted by d,

Then $u_1 = 2 = a$

$u_2 = 5 = a + d$

$u_3 = 8 = a + 2d$

$u_4 = 11 = a + 3d$

$u_5 = 14 = a + 4d$

$\vdots$

$u_n = a + (n-1)d$ ⟵ Last term

Thus any specific term in an Arithmetic progression is obtained by the formula.

$$\boxed{U_n = a + (n-1)d}$$

where a is the first term

d is the common difference

n is the term we are looking for.

Example: 1

Find the 8^{th}, 10^{th} and 20^{th} term in the Serie 1, 3, 5, 7, - - -

soln

8^{th} term: first term $a = 1$

common difference $d = 2$.

$\therefore$ n is 8

$\therefore$ using $U_n = a + (n-1)d$

$U_8 = 1 + (8-1)(2)$

$U_8 = 1 + 14$

$U_8 = 15$.

$\therefore$ The 8^{th} term is 15

10^{th} term: $n=10$, $a=1$, $d=2$.

$U_n = a+(n-1)d$

$U_{10} = 1+(10-1)2$

$U_{10} = 1+18$

$U_{10} = 19$

$\therefore$ The 10^{th} term is 19.

20^{th} term: $n=20$, $a=1$, $d=2$.

$U_n = a+(n-1)d$

$U_{20} = 1+(20-1)2$

$U_{20} = 1+38$

$\therefore U_{20} = 39$.

$\therefore$ The 20^{th} term is 39.

$\boxed{\text{Exercise 1a}}$

1. State the first term (a) and the Common difference (d) in the Series below

a) 8, 11, 14, 17, ---

b) −8, −7, −6, −5, ---

c) $6\frac{1}{4}$, $6\frac{3}{4}$, $7\frac{1}{4}$, $7\frac{3}{4}$, ---

2. Find 30^{th}, 50^{th} and 80^{th} term in each of the ~~above~~ Series in 1) above.

3. Given that 100 is a term in the Serie 2, 4, 6, --- what is the position number of 100.

Sum of terms in Arithmetic progression

Suppose we wish to add agiven number of terms in a serie $a, a+d, a+2d, ---, a+(n-1)d$. We say

$$S_n = a + (a+d) + (a+2d) + --- + (a+(n-1)d) \quad \text{--- (1)}$$

Rewriting in reverse order

$$S_n = (a+(n-1)d) + a+(n-2)d + --- + (a+d) + a \quad \text{--- (11)}$$

Adding (1) and (11) gives

$$2S_n = (2a+(n-1)d) + (2a+(n-1)d) + --- + [2a+(n-1)d]$$

$$2S_n = n(2a+(n-1)d)$$

$$\boxed{\therefore S_n = \frac{n}{2}(2a+(n-1)d)}$$

Notice that the above formula can be written as $S_n = \frac{n}{2}(a + \underbrace{a+n(n-1)d}_{U_n \text{ (last term)}})$

but $a+n(n-1)d$ is the last term thus denoting the last term by l the formula becomes

$$S_n = \frac{n}{2}(a+l) \text{ where } l \text{ is the last term.}$$

Generally for Arithmetic progression
n^{th} term is given by $U_n = a+(n-1)d$
Sum of n number of terms is given by
$$S_n = \frac{n}{2}(2a+(n-1)d)$$

Examples: 2. Find the Sum of the first 20 terms in the Serie 1, 3, 5, 7, ---.

Soln.

$a = 1, d = 2, n = 20$

using $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}(2 \times 1 + (20-1)2)$

$S_{20} = 10(2 + 38)$

$\therefore S_{20} = 400$

$\therefore$ The Sum of the first 20 terms is 400.

Example 3: Find the Sum of the terms in the Serie: 2, 4, 6, ---, 146

Soln:

Notice that the last term l is 146

$\therefore a + (n-1)d = 146$. but $a = 2, d = 2$

$\therefore 2 + (n-1)2 = 146 \Rightarrow n = 73$ hence there are 73 terms

using the second formula $S_n = \frac{n}{2}(a + l)$

$S_n = \frac{n}{2}(a + l)$ but the Serie has 73 terms

$\therefore S_{73} = \frac{73}{2}(2 + 146)$

$S_{73} = 5476$

OR

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{73} = \frac{73}{2}(2 \times 2 + (73-1)2)$

$S_{73} = \frac{73}{2}(4 + 144)$

$\therefore S_{73} = 5476$

$\therefore$ The Sum of the terms is 5476.

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Example:

How many terms are in the Serie 100, 95, 90, 85, ---, −20.

Soln:

Note that $a = 100$, $d = -5$

last term $l = -20$

using $l = a + (n-1)d$

$-20 = 100 + (n-1) \cdot -5$

$-20 = 100 - 5n + 5$

$5n = 100 + 5 + 20$

$5n = 125$

$n = 25$

∴ There are 25 terms in that Serie.

Exercise 1b)

1. a) How many terms are in ~~a~~ the series below

i) 2, 4, 6, --- 80.

ii) 4, 9, 14, 19, ---, 800.

iii) 100, 92, 84, ---, −60.

b) Find the Sum of the first 50 terms of the series

i) 2, 4, 6, 8, ---

ii) 1, 3, 5, 7, ---

~~iii) $5\frac{1}{4}$, $4\frac{1}{2}$, $3\frac{3}{4}$~~

iii) $5\frac{1}{4}$, $4\frac{1}{2}$, $3\frac{3}{4}$, ---

(iv) 80, 75, 70, ---

Finding first term a or common difference d or both.

Sometimes we may be asked to find the first term a or the common difference d or both when given other information about the serie.

Examples:

1. In an A.P the fifth term is 8 and the nineth term is 14. Find the first term a and the common difference d.

Soln.

Given

$U_5 = 8$ and $U_9 = 14$.

using $U_n = a + (n-1)d$.

$U_5 = a + (5-1)d$.

$8 = a + 4d$

$\Rightarrow a + 4d = 8$ — (i).

also

$U_9 = a + (9-1)d$

$U_9 = a + 8d$.

$14 = a + 8d$

$\Rightarrow a + 8d = 14$ — (ii).

Solving (ii) and (i) simultaneously

eqn (ii) − eqn (i)

$$\begin{array}{r} a + 8d = 14 \\ - \quad a + 4d = 8 \\ \hline 4d = 6 \end{array}$$

$\Rightarrow d = 3/2$

Sub $d = 3/2$ in eqn (i)

$a + 4(3/2) = 8$

$a = 2$.

∴ The first term is 2 and the common difference is $3/2$

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2. In an Arithmetic progression the third term is 7.5 and the tenth term is 11. find the first term, the common differfence and the sum of the first 20 terms of the serie.

Soln:

Given $U_3 = 7.5$, and $U_{10} = 11$.

But $U_n = a + (n-1)d$.

$U_3 = a + (3-1)d$.

$7.5 = a + 2d$.

$\Rightarrow a + 2d = 7.5$ —— ①.

also, $U_{10} = a + (10-1)d$.

$11 = a + (10-1)d$

$\Rightarrow a + 9d = 11$ —— ⑪.

Solve ⑪ $ ① simultenously

eqn ⑪ − eqn ①

$$\begin{array}{r} a + 9d = 11 \\ - \quad a + 2d = 7.5 \\ \hline \frac{7d}{7} = \frac{3.5}{7} \end{array}, \Rightarrow d = 0.5$$

Sub $d = 0.5$ in eqn ①

$a + 2(\frac{1}{2}) = 7.5 \Rightarrow a = 6.5$

Hence the first term is 6.5 and the common $d = 0.5$

Sum of first 20 terms: $n = 20, a = 6.5, d = 0.5$

$S_{20} = \frac{n}{2}(2a + (n-1)d) = \frac{20}{2}(2(6.5) + (20-1)(0.5))$

∴ The sum of the first 20 terms is 225

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3. In An Arithmetic Progression the sum of the forth and Sixth term is 22 and the difference between the twelveth and the Seveth term is 10. Find the first term and the common difference

Soln:

Given; $U_4 + U_6 = 22$ also $U_{12} - U_7 = 10$

but $U_4 = a+(4-1)d$

$U_4 = a+3d$

$U_6 = a+(6-1)d$

$U_6 = a+5d$

$\therefore U_4 + U_6 = (a+3d)+(a+5d) = 22$

$2a + 8d = 22$

$a + 4d = 11$ — ①

also, $U_{12} = a+(12-1)d$ and $U_7 = a+(7-1)d$

$= a+11d$ $\quad U_7 = a+6d$

$\therefore U_{12} - U_7 = (a+11d) - (a+6d) = 10$

$5d = 10$

$\Rightarrow d = 2.$

Sub $d=2$ in eqn ①

$a+4d = 11$

$a+4(2) = 11$

$a = 3.$

$\therefore$ The first term is 3 and the Common difference is 2.

Exercise 1c)

a) In an Arithmetic Progression the fifth term is 9 and the tenth term is 19. Find the first term and the common difference.

b) The Sum of the forth and the Seveth term is 18 and the Sum of the third and the nineth term is 20. Find the first term & the common difference